2.5b Uses for Determinants

We can use determinants for more than just Cramer's Rule.

Area of a Triangle The area of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is given by:

choose the sign that makes the area positive

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

all the x-coordinates of the vertices

3rd column is always all 1's.

all the y-coordinates of the vertices

Example 1 Find the area of a triangle whose vertices are (1, 0), (2, 2), and (4, 3)

$8 + 3 + 0 = 11$

$$\text{Area} = \pm\frac{1}{2}\left|\begin{array}{ccc|cc} 1 & 0 & 1 & 1 & 0 \\ 2 & 2 & 1 & 2 & 2 \\ 4 & 3 & 1 & 4 & 3 \end{array}\right.$$

$2 + 0 + 6 = 8$

$8 - 11 = -3$

$-\frac{1}{2}\cdot(-3) = \frac{3}{2}u^2$ or $1.5\, u^2$

Test for Collinear Points

Three points are collinear ~~if and only if~~:

Same determinant used to find the area of a triangle

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Example 2 Determine whether the points (-2, -2), (1, 1), and (7, 5) are collinear.

$7 - 10 - 2 = -5$

$$\left|\begin{array}{ccc} -2 & -2 & 1 \\ 1 & 1 & 1 \\ 7 & 5 & 1 \end{array}\right| \begin{array}{cc} -2 & -2 \\ 1 & 1 \\ 7 & 5 \end{array}$$

$-2 - 14 + 5 = -11$

$-11 + {}^{-}5 = -6$

$-6 \neq 0$ so the points are not colinear.

$\text{Area} = -\frac{1}{2}(-6) = \underline{3\,u^2}$

If the points are collinear, you may want to find the equation of the line that they are on. Once again, you can use determinants.

~~Two Point Form of the Equation of a Line~~ (standard form $Ax + By + C = 0$)

An equation for the line passing through the distinct points, (x_1, y_1) and (x_2, y_2) is given by evaluating:

variables

x-coordinates

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

y-coordinates

Works for every equation! (even special lines)

Ex 3 Find an equation for the line passing through the two points (2, 4) and (-1, 3).

$(-4 + 3x + 2y)$

$$\left|\begin{array}{ccc} x & y & 1 \\ 2 & 4 & 1 \\ -1 & 3 & 1 \end{array}\right| \begin{array}{cc} x & y \\ 2 & 4 \\ -1 & 3 \end{array}$$

$(4x - 1y + 6)$

$(4x - 1y + 6) - 1(-4 + 3x + 2y)$

$4x - y + 6 + 4 - 3x - 2y = 0$

$x - 3y + 10 = 0$