

2.5b Uses for Determinants

We can use determinants for more than just Cramer's Rule.

Area of a Triangle The area of a triangle with vertices (x_1, y_1) , (x_2, y_2) , and (x_3, y_3) is given by:

Choose the sign that makes the area positive

$$\text{Area} = \pm \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

all the x-coordinates of the vertices
 ← 3rd column is always all 1's.

all the y-coordinates of the vertices

Example 1 Find the area of a triangle whose vertices are $(1, 0)$, $(2, 2)$, and $(4, 3)$

$$\text{Area} = \pm \frac{1}{2} \begin{vmatrix} 1 & 0 & 1 \\ 2 & 2 & 1 \\ 4 & 3 & 1 \end{vmatrix}$$

$8 + 3 + 0 = 11$

$2 + 0 + 6 = 8$

$8 - 11 = -3$
 $-\frac{1}{2} \cdot (-3) = \frac{3}{2} u^2$
 or $1.5 u^2$

Test for Collinear Points

Three points are collinear if and only if:

Same determinant used to find the area of a triangle

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Example 2 Determine whether the points $(-2, -2)$, $(1, 1)$, and $(7, 5)$ are collinear.

$$\begin{vmatrix} -2 & -2 & 1 \\ 1 & 1 & 1 \\ 7 & 5 & 1 \end{vmatrix} = \begin{matrix} 7 - 10 - 2 = -5 \\ -2 - 2 \\ -2 - 14 + 5 = -11 \\ -11 + 5 = -6 \\ -6 \neq 0 \text{ so the points are not collinear.} \end{matrix}$$

$$\text{Area} = -\frac{1}{2}(-6) = \underline{3} \text{ u}^2$$

If the points are collinear, you may want to find the equation of the line that they are on. Once again, you can use determinants.

Two Point Form of the Equation of a Line (standard form $Ax + By + C = 0$)

An equation for the line passing through the distinct points, (x_1, y_1) and (x_2, y_2) is given by evaluating:

variables \rightarrow

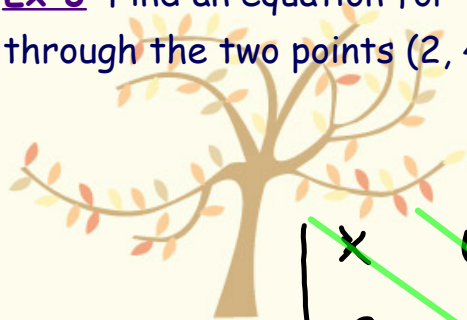
$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

x-coordinates \rightarrow

y-coordinates \rightarrow

Works for every equation! (even special lines)

Ex 3 Find an equation for the line passing through the two points $(2, 4)$ and $(-1, 3)$.



$$\begin{vmatrix} x & y & 1 \\ 2 & 4 & 1 \\ -1 & 3 & 1 \end{vmatrix} = \begin{vmatrix} x & y \\ 2 & 4 \\ -1 & 3 \end{vmatrix}$$

$$(-4 + 3x + 2y)$$

$$(4x - 1y + 6)$$

$$(4x - 1y + 6) - 1(-4 + 3x + 2y)$$

$$4x - y + 6 + 4 - 3x - 2y = 0$$

$$x - 3y + 10 = 0$$